

YEAR 7 — LINES AND ANGLES

Constructing, measuring and using geometric notation

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use letter and labeling conventions
- Draw and measure line segments and angles
- Identify parallel and perpendicular lines
- Recognise types of triangle
- Recognise types of quadrilateral
- Identify polygons
- Construct triangles (SAS, SSS, ASA)
- Draw Pie charts

Keywords

- Polygon:** A 2D shape made with straight lines
- Scalene triangle:** a triangle with all different sides and angles
- Isosceles triangle:** a triangle with two angles the same size and two sides the same size
- Right-angled triangle:** a triangle with a right angle
- Frequency:** the number of times a data value occurs
- Sector:** part of a circle made by two radii touching the centre
- Rotation:** turn in a given direction
- Protractor:** equipment used to measure angles
- Compass:** equipment used to draw arcs and circles

Letter and labeling convention

The letter in the middle is the angle
The arc represents the angle

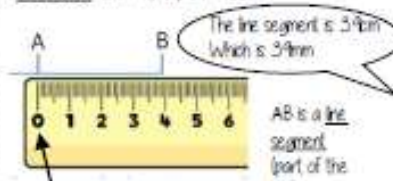


Angle Notation: three letters ABC
This is the angle at B = 113°

Line Notation: two letters EC
The line that joins E to C

Draw and measure line segments

Conversions: 1cm = 10mm, 1m = 100cm



Make sure the start of the line is at 0

Angles as measures of turn

East to South is a quarter turn clockwise

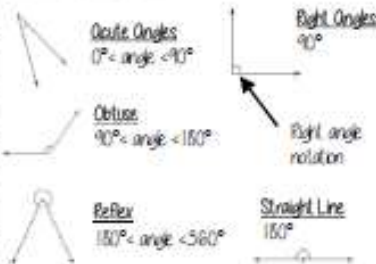
Quarter Turn 90°
Clockwise

Half Turn 180°
Anti-Clockwise

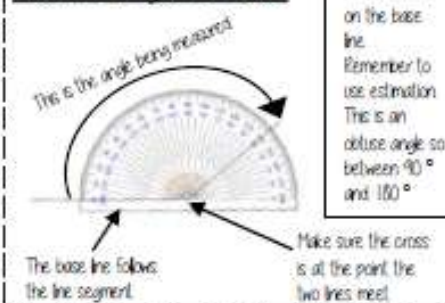
Three-quarter Turn 270°
Anti-Clockwise

Full Turn 360°

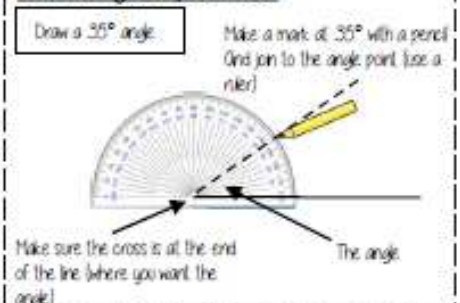
Classify angles



Measure angles to 180°



Draw angles up to 180°



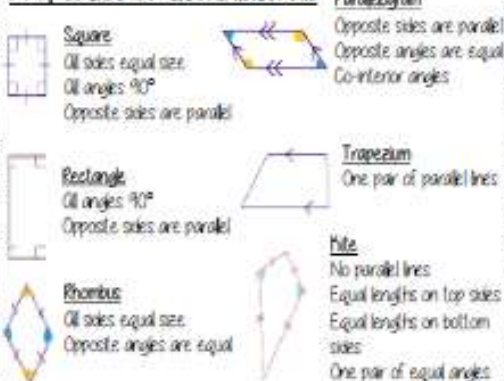
Parallel and Perpendicular lines



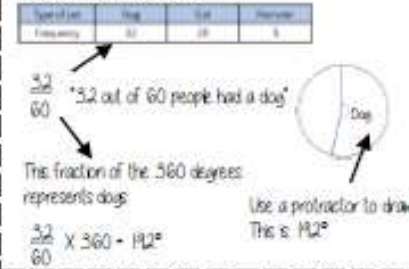
Angles over 180°



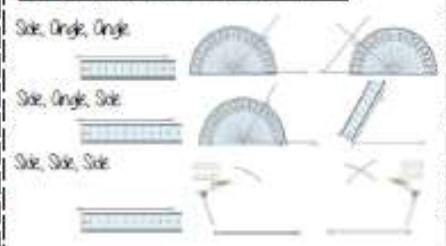
Properties of Quadrilaterals



Draw Pie Charts



SAS, SSS, ASA constructions



Polygons

3 - Triangle	5 - Pentagon	8 - Octagon
4 - Quadrilateral	6 - Hexagon	9 - Nonagon
	7 - Heptagon	10 - Decagon

If all the sides and angles are the same, it is a regular polygon

YEAR 7 — LINES AND ANGLES

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Geometric reasoning

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand/use the sum of angles at a point
- Understand/use the sum of angles on a straight line
- Understand/use equality of vertically opposite angles
- Know and apply the sum of angles in a triangle
- Know and apply the sum of angles in a quadrilateral

Keywords

Vertically Opposite: angles formed when two or more straight lines cross at a point

Interior Angles: angles inside the shape

Sum total: add all the interior angles together

Convex Quadrilateral: a four-sided polygon where every interior angle is less than 180°

Concave Quadrilateral: a four-sided polygon where one interior angle exceeds 180°

Polygon: a 2D shape made with straight lines

Scalene triangle: a triangle with all different sides and angles

Isosceles triangle: a triangle with two angles the same size and two sides the same size

Right-angled triangle: a triangle with a right angle

Sum of angles at a point

The sum of angles around a point is 360°



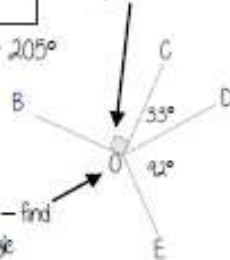
Find angle BOE

$$90^\circ + 33^\circ + 92^\circ = 205^\circ$$

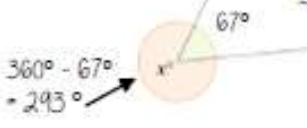
$$360^\circ - 205^\circ$$

$$\text{BOE} = 155^\circ$$

Angle notation — 90°

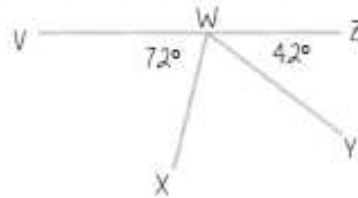


Angle notation — find this missing angle



Sum of angles on a straight line

Adjacent angles that share a common point on a line add up to 180°

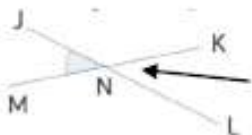


Find angle XWY

$$72^\circ + 42^\circ = 114^\circ$$

$$180^\circ - 114^\circ = 66^\circ$$

Vertically opposite angles

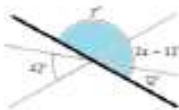


Angle JNM is vertically opposite to angle FNL

$$\text{JNM} = \text{FNL}$$

Vertically opposite angles are the same

Other angle rules still apply. Look for straight line sums and angles around a point.

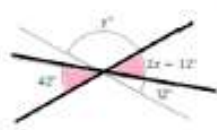


Form equations with information from diagrams

$$2x - 12 = 42$$

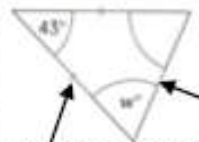
$$2x = 54$$

$$x = 27^\circ$$



Sum of angles in triangles

Sum of interior angles in a triangle = 180°



The two base angles will be the same size

Look at triangle notation. This indicates an isosceles triangle

$$\therefore 180 - 43 = 137$$

$$137 \div 2 = 68.5^\circ$$

A triangle can only have ONE right angle



Have a go!

Tearing the corners from triangles forms a straight line which is therefore 180°

Sum of angles in quadrilaterals

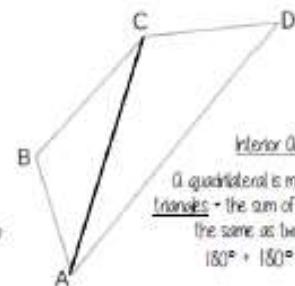
Sum of interior angles in a quadrilateral = 360°



Convex Quadrilateral
Concave Quadrilateral



Interior angles are those that make up the perimeter (outline) of the shape

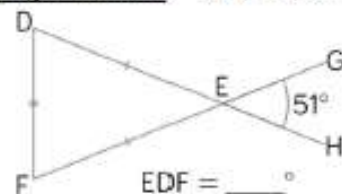


Interior Angles

A quadrilateral is made up of two triangles — the sum of interior angles is the same as two triangles
 $180^\circ + 180^\circ = 360^\circ$

Angle Problems

Split up the problem into chunks and explain your reasoning at each point using angle notation



1. Angle DEF = 51° because it is a vertically opposite angle DEF = GEH

2. Triangle DEF is isosceles (triangle notation) \therefore EDF = EFD and the sum of interior angles is 180°
 $180^\circ - 51^\circ = 129^\circ$ $129^\circ \div 2 = 64.5^\circ$

3. Angle EDF = 64.5°

Keep working out clear and notes together

YEAR 7 — REASONING WITH NUMBER

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Developing number sense

What do I need to be able to do?

By the end of this unit you should be able to

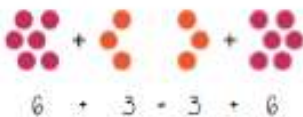
- Know and use mental addition/ subtraction
- Know and use mental multiplication/ division
- Know and use mental arithmetic for decimals
- Know and use mental arithmetic for fractions
- Use factors to simplify calculations
- Use estimation to check mental calculations
- Use number facts
- Use algebraic facts

Keywords

- Commutative:** changing the order of the operations does not change the result
- Associative:** when you add or multiply you can do so regardless of how the numbers are grouped
- Dividend:** the number being divided
- Divisor:** the number we divide by
- Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
- Equation:** a mathematical statement that two things are equal
- Quotient:** the result of a division

Mental methods for addition/ subtraction

Addition is commutative



The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction

Mental methods for multiplication/ division

Multiplication is commutative



The order of multiplication does not change the result

Partitioning can help multiplication

$$24 \times 6 = 20 \times 6 + 4 \times 6 \\ = 120 + 24 \\ = 144$$

Division is not associative

Chunking the division can help $4000 \div 25$
"How many 25's in 100" then how many chunks of that in 4000

Mental methods for decimals

Multiplying by a decimal < 1 will make the original value smaller. eg $\times 0.1 = \div 10$

Methods for multiplication 12×0.03

$$\begin{array}{r} 12 \times 3 = 36 \\ 12 \times 3 = 36 \\ 12 \times 0.3 = 3.6 \\ 12 \times 0.03 = 0.36 \end{array} \quad \begin{array}{r} 12 \times 3 = 36 \\ + 10 \quad + 100 \quad + 1000 \\ 12 \times 0.03 = 0.36 \end{array}$$

Methods for division $15 \div 0.05$

Multiply by powers of 10 until the divisor becomes an integer

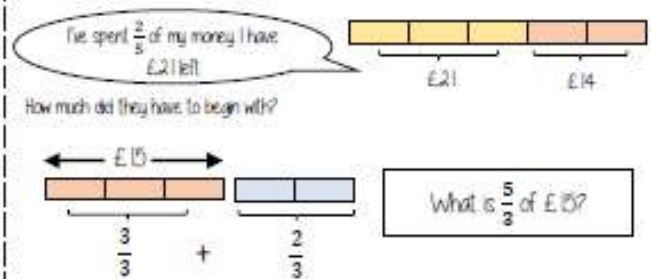
$$\begin{array}{r} \times 100 \\ 15 \div 0.05 = 300 \\ \times 100 \\ 150 \div 5 = 30 \end{array}$$

Methods for addition $2.3 + 2.4$

$$\begin{array}{r} 2 + 2 = 4 \\ 0.3 + 0.4 = 0.7 \\ 4 + 0.7 = 4.7 \end{array}$$

Mental methods for fractions

Use bar models where possible



Using factors to simplify calculations

$$30 \times 16$$

$$10 \times 3 \times 4 \times 4$$

$$10 \times 3 \times 2 \times 8$$

$$2 \times 5 \times 3 \times 2 \times 2 \times 2 \times 2$$

$$16 \times 10 \times 3$$

Multiplication is commutative
Factors can be multiplied in any order

Estimation

Estimations are useful — especially when using fractions and decimals to check if your solution is possible

Most estimations round to 1 significant figure

Estimations are useful — especially when using fractions and decimals to check if your solution is possible

$$210 \times 899 < 1200$$

This is true because even if both numbers were rounded up, they would reach $300 + 900$

The correct estimation would be $200 \times 900 = 1100$

Number facts

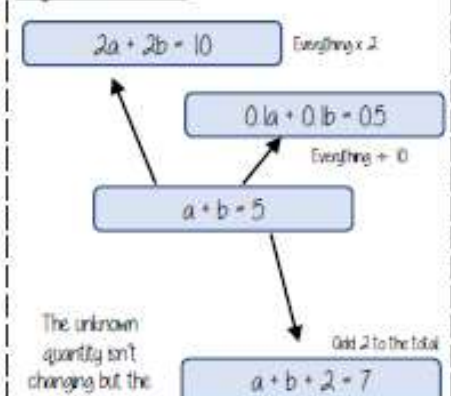
Use $124 \times 5 = 620$

For multiplication, each value that is multiplied or divided by powers of 10 needs to happen to the result

$$620 \div 124 = 50$$

For division you must consider the impact of the divisor becoming smaller or bigger
Smaller — the answer will be bigger (it is being shared into less parts)
Bigger — the answer will be smaller (it is being shared into more parts)

Algebraic facts



The unknown quantity isn't changing but the variables change what is done to give the result

YEAR 7 — REASONING WITH NUMBER

Sets and probability

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What do I need to be able to do?

By the end of this unit you should be able to:

- Identify and represent sets
- Interpret and create Venn diagrams
- Understand and use the intersection of sets
- Understand and use the union of sets
- Generate sample spaces for single events
- Calculate the probability of a single event
- Understand and use the probability scale

Keywords

Set: collection of things
Element: each item in a set is called an element
Intersection: the overlapping part of a Venn diagram (\cap and n)
Union: two ellipses that join (\cup)
Mutually Exclusive: events that do not occur at the same time
Probability: likelihood of an event happening
Bias: a built-in error that makes all values wrong (unequal) by a certain amount, e.g. a weighted die
Fair: there is zero bias, and all outcomes have an equal likelihood
Random: something happens by chance and is unable to be predicted

Identify and represent sets

The universal set has this symbol ξ — this means EVERYTHING in the Venn diagram is in this set.

A set is a collection of things — you write sets inside curly brackets { }

ξ = {the numbers between 1 and 50 inclusive}

My sets can include every number between 1 and 50 including those numbers

A = {Square numbers}

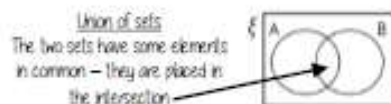
A = {1, 4, 9, 16, 25, 36, 49}

All the numbers in set A are square number and between 1 and 50

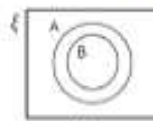
Interpret and create Venn diagrams



Mutually exclusive sets
 The two sets have nothing in common
 No overlap



Union of sets
 The two sets have some elements in common — they are placed in the intersection

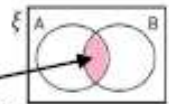


Subset
 All of set B is also in Set A so the ellipse fits inside the set.

The box
 Around the outside of every Venn diagram will be a box. If an element is not part of any set it is placed outside an ellipse but inside the box.

Intersection of sets

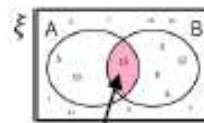
Elements in the intersection are in set A AND set B



The notation for this is $A \cap B$

ξ = {the numbers between 1 and 15 inclusive}

A = {Multiples of 5} B = {Multiples of 3}

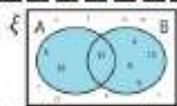


The element in $A \cap B$ is 15

In this example there is only one number that is both a multiple of 3 and a multiple of 5 between 1 and 15

Union of sets

Elements in the union could be in set A OR set B



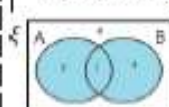
ξ = {the numbers between 1 and 15 inclusive}

A = {Multiples of 5} B = {Multiples of 3}

The elements in $A \cup B$ are 5, 10, 15, 3, 9, 6, 12

There are 7 elements that are either a multiple of 5 OR a multiple of 3 between 1 and 15

The notation for this is $A \cup B$



This Venn shows the number of elements in each set

Sample space — for single events



A sample space for rolling a six-sided die is: $S = \{1, 2, 3, 4, 5, 6\}$



A sample space for this spinner is: $S = \{\text{Pink, Blue, Yellow}\}$

You only need to write each element once in a sample space diagram

- A Sample space represents a possible outcome from an event
- They can be interpreted in a variety of ways because they do not tell you the probability

Probability of a single event



Probability = $\frac{\text{number of times event happens}}{\text{Total number of possible outcomes}}$

$P(\text{Blue}) = \frac{4}{10}$ ← There are 4 blue sectors
 $\frac{2}{5}$ ← There are 10 sectors overall

Probability can be a fraction, decimal or percentage value

$\frac{4}{10}$ $\frac{40}{100}$ $= 0.40$ $= 40\%$

Probability is always a value between 0 and 1

The probability scale

Impossible 0 or 0% Even chance 0.5, $\frac{1}{2}$ or 50% Certain 1 or 100%

The more likely an event the further up the probability it will be in comparison to another event (it will have a probability closer to 1)



There are 2 pink and 3 yellow balls, so they have the same probability

There are 5 possible outcomes
 So 5 intervals on the scale, each interval value is $\frac{1}{5}$

Sum of probabilities

Probability is always a value between 0 and 1



The probability of getting a blue ball is $\frac{2}{5}$
 ∴ The probability of NOT getting a blue ball is $\frac{3}{5}$
 The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$



YEAR 7 — REASONING WITH NUMBER

Prime numbers and Proof

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What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number

Prime: an integer with only 2 factors

Conjecture: a statement that might be true (based on reasoning) but is not proven

Counterexample: a special type of example that disproves a statement

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

HCF: highest common factor (biggest factor two or more numbers share)

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3

3, 6, 9, 12, 15, ...

3x, 6x, 9x, ...

The list continues and doesn't end

Non-example of a multiple

45 is not a multiple of 3

because it is 3×15

Not an integer

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Factors

Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

Factors and expressions

6x x 1 OR 6 x x

Factors of 6x
6, x, 1, 6x, 2x, 3, 3x, 2

2x x 3

3x x 2

The number itself is always a factor

Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number
The only even prime number

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...

Square and triangular numbers

Square numbers



Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64, ...

Triangular numbers

Representations are useful — an extra counter is added to each new row



1, 3, 6, 10, 15, 21, 28, 36, 45, ...

Common factors and HCF

Common factors are factors two or more numbers share

HCF — Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

1 is a common factor of all numbers

Common factors
(factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM — Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match



Comparing fractions

$\frac{3}{5}$ and $\frac{7}{10}$

Compare fractions using a LCM denominator

$\frac{6}{10}$ and $\frac{7}{10}$

Conjectures and counterexamples

Conjecture

1, 2, 4, ...
The numbers in this sequence are doubling each time.

Counterexamples



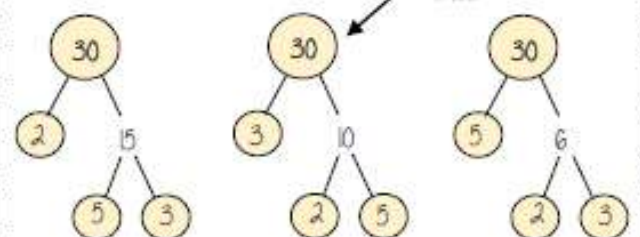
This sequence isn't doubling it is adding 2 each time.

A pattern that is noticed for many cases

Only one counterexample is needed to disprove a conjecture

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

$30 = 2 \times 3 \times 5$

Multiplication of prime factors

Using prime factors for predictions

e.g. 60: 30×2 or $2 \times 3 \times 5 \times 2$

150: 30×5 or $2 \times 3 \times 5 \times 5$